basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2010

MEMORANDUM

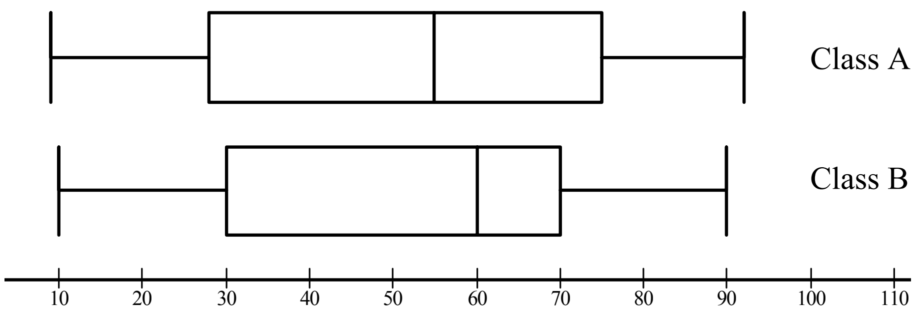
MARKS: 150

This memorandum consists of 33 pages.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent Accuracy applies in **ALL** aspects of the marking memorandum.

QUESTION 1

1.1	The Data Set 9 14 14 19 21 23 $\overset{Q_1}{\downarrow}$ 33 35 37 37 42 45 $\overset{Q_2}{\downarrow}$ 55 56 57 59 68 75 $\overset{Q_3}{\downarrow}$ 75 75 77 78 80 81 92 Min = 9 Max = 92 Q_3 = Upper Quartile = 75 Q_1 = Lower Quartile = 28 (6^{th} number is 23. 7^{th} number is 33. The number in position 6,25 is $23 + \frac{1}{4}(33 - 23) = 25{,}5$) (Accept $Q_1 = 25{,}5$) Q_2 = Median = 55 Five number summary is (9 ; 28 ; 55 ; 75 ; 92) OR (9 ; 25,5 ; 55 ; 75 ; 92)	✓ min & max ✓ upper quartile ✓ lower quartile ✓ median (4) **Note:** Penalty 1 for not labelling or writing the numbers in ascending order
1.2		✓ box ✓ whiskers (2)

Note:
If the candidate combines the answer of 1.1 and 1.2 by drawing the correct box and whisker diagram in 1.2 and writes the numbers on the diagram (i.e. we mark question 1.1 and 1.2 in conjunction with one another), **max 5 / 6 marks for question 1.1 and 1.2**

If candidate writes numbers randomly in 1.1 and draws the box and whisker diagram correctly but without indicating the numbers on the diagram, then **max 5 / 6 marks for question 1.1 and 1.2**

If candidate just draws the box and whisker in 1.2 and does not indicate values on it or answers 1.1, the **max 2 / 6 marks**

If the candidate draws two diagrams (one in the answer book and one on the diagram sheet), mark the one on the **DIAGRAM SHEET**.

1.3	Class B Class B performed better because half of the learners got above 60% whilst half of Class A got more than 55%. Class B performed better because half of the learners got above 60% whilst half of Class A got less than 55%. Median of Class B > Median of Class A **OR** Class B Class B is skewed more to the left than Class A is. **OR** Class A 25% of class scored 75% or more in Class A while 25% of the class scored 70% or more in Class B. Highest Mark in Class A > Highest Mark in Class B. **Note:** If candidate answers: Cannot determine the class that does better because we have insufficient information as we do not know where the marks are clustered. **Max 1 / 3** **Note:** If candidate just answers Class A or Class B and there are no reasons, then **0 / 3 marks**	✓ Class B ✓✓ median Class B > Median Class A (3) ✓ Class B ✓✓ Class B skewed more left than A (3) ✓ Class A ✓ highest A > highest B 25% of A above 75% and 25% of B above 70% (2) **[9]**

QUESTION 2

2.1

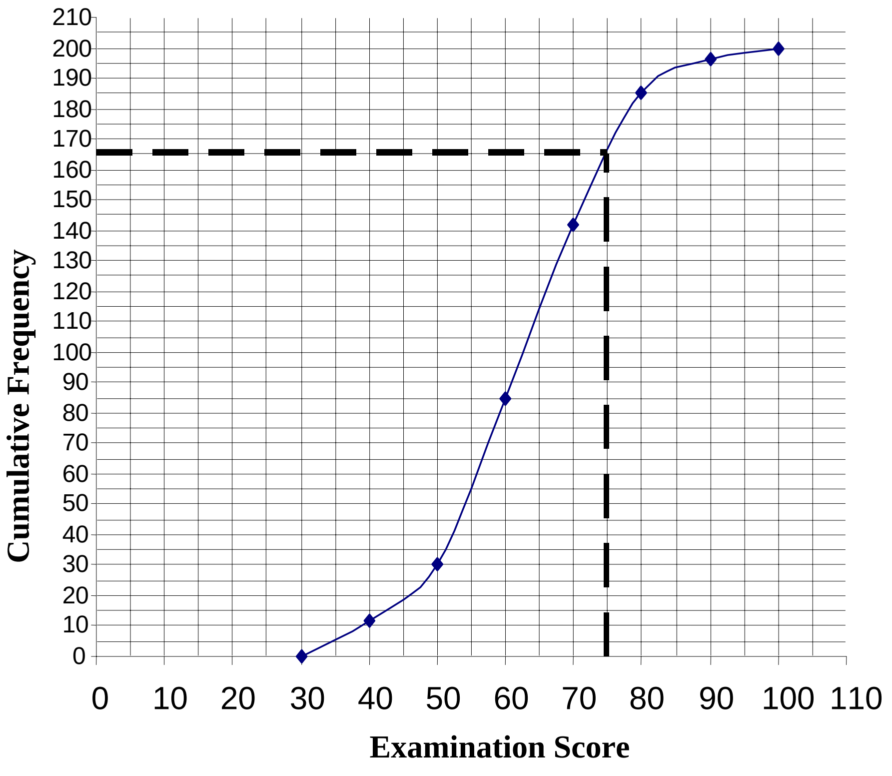
EXAMINATION SCORE (x)	FREQUENCY	CUMULATIVE FREQUENCY
$30 \le x < 40$	12	12
$40 \le x < 50$	18	30
$50 \le x < 60$	55	85
$60 \le x < 70$	57	142
$70 \le x < 80$	43	185
$80 \le x < 90$	11	196
$90 \le x < 100$	4	200

✓ first 3 values

✓ last 4 values
(2)

2.2

Cumulative frequency graph of examination scores



✓shape (points must not be joined a straight line with a ruler)

✓ grounding point (30 ; 0)

✓ using the upper limit

✓ using cumulative frequencies

✓if 4 or more points plotted correctly

(5)

Note:
If learners plot the midpoint of the interval and the cumulative frequency **max 1 / 5 marks** for shape

If the candidates plot the lower limit and the cumulative frequency **max 1 / 5 marks**

OR

$$\frac{142+185}{2} = 163{,}5$$

$$200 - 163{,}5 = 36{,}5$$

Note:
Accept any one of 34, 35 or 36

✓ answer
(1)
[8]

QUESTION 3

<table>
<tr><td>3.1</td><td>Mean
$= \dfrac{217+211+221+239+144+161+168+185+265+249+160+184}{12}$
$= \dfrac{2404}{12}$
$= 200{,}33$

Note:
Penalty 1 for incorrect rounding</td><td>✓ $\dfrac{2404}{12}$
✓ answer
(2)
Answer only: Full marks</td></tr>
<tr><td>3.2</td><td>By means of a calculator:
$\sigma = 37{,}37$

Note:
No penalty for incorrect decimal places

Accept 37

OR

Pen and paper method:
mean ($\bar{x}$) $= 200{,}33$

<table>
<tr><th>x</th><th>$x-\bar{x}$</th><th>$(x-\bar{x})^2$</th></tr>
<tr><td>217</td><td>16,67</td><td>277,889</td></tr>
<tr><td>211</td><td>10,67</td><td>113,848</td></tr>
<tr><td>221</td><td>20,67</td><td>427,248</td></tr>
<tr><td>239</td><td>38,67</td><td>1495,368</td></tr>
<tr><td>144</td><td>-56,33</td><td>3173,0689</td></tr>
<tr><td>161</td><td>-39,33</td><td>1546,848</td></tr>
<tr><td>168</td><td>-32,33</td><td>1045,228</td></tr>
<tr><td>185</td><td>-15,33</td><td>235,008</td></tr>
<tr><td>265</td><td>64,67</td><td>4182,208</td></tr>
<tr><td>249</td><td>48,67</td><td>2368,768</td></tr>
<tr><td>160</td><td>-40,33</td><td>1626,508</td></tr>
<tr><td>184</td><td>-16,33</td><td>266,668</td></tr>
<tr><td colspan="2">SUM</td><td>16758,666</td></tr>
</table>

$\sigma = \sqrt{\dfrac{16758{,}6668}{12}} = 37{,}37$</td><td>✓✓✓ answer
(3)

✓total

✓ substitution
✓answer
(3)</td></tr>
<tr><td>3.3</td><td>200,33+1(37,37)
= 237,70 litres

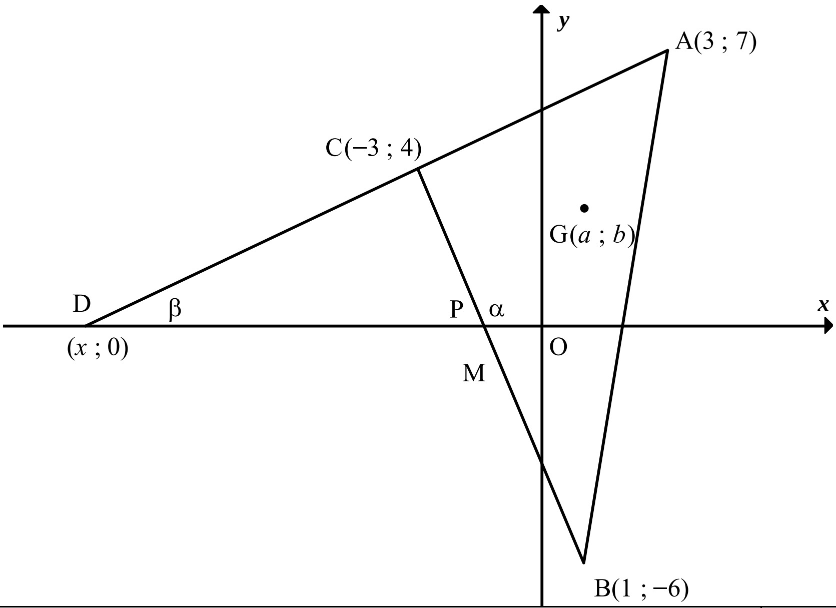
Accept any number between and including 237 and 238 litres.

Note:
If candidate answers
200,33 – 1(37,37) = 162,96 litres
Max 1 / 2 marks</td><td>✓method
✓answer
(2)
[7]</td></tr>
</table>

QUESTION 4

4.1	Fly High	✓ answer (1)
4.2	$\frac{5120}{1000}\times 7{,}9$ $= 40{,}45$ **OR** $\frac{5120}{1000}\times 8$ $= 40{,}96$ **OR** 8×5 $= 40$ Yes	✓ calculation that leads to 40 (1)
4.3	Yes. The data points suggest a straight line fit with negative gradient but Fly-High will have to be an outlier. **OR** Yes. Weak negative correlation. (r = – 0,2128075984) **Note:** If the candidate indicates "Best Air" and/or "Best Fly" and/or "Alpha" have high on time arrivals and low lost luggage **max 1 / 2 marks**	✓Yes ✓ negative gradient (2) ✓Yes ✓ negative correlation (2)
4.4	Alpha, 70% on-time arrival and least luggage loss **OR** Best Air, best on time arrival	✓ Name of company ✓correct justification (2) **[6]**

QUESTION 5



5.1.1	$m_{AD} = m_{AC}$ $= \frac{7-4}{3-(-3)}$ $= \frac{3}{6}$ $= \frac{1}{2}$ **OR** $m_{AD} = m_{AC}$ $= \frac{4-7}{-3-(3)}$ $= \frac{-3}{-6}$ $= \frac{1}{2}$ **Note:** If candidate gives $m_{AD} = \frac{7}{3-x}$ then **1/2 marks**	✓ substitution of A and C into correct formula ✓ answer (2)
5.1.2	$m_{BC} = \frac{-6-4}{1-(-3)}$ $= \frac{-10}{4}$ $= \frac{-5}{2}$ **OR** $m_{BC} = \frac{4-(-6)}{-3-(1)}$ $= \frac{10}{-4}$ $= \frac{-5}{2}$	✓ answer (1)
5.2	$m_{AD} = \frac{1}{2} = \tan C\hat{D}O$ $C\hat{D}O = 26{,}56505....°$ $m_{BC} = \frac{-5}{2} = \tan\alpha$ $\alpha = 111{,}814\,095°$ $D\hat{C}B = 111{,}8014095....° - 26{,}56505....°$ $= 85{,}236359°$ $= 85{,}24°$ $\approx 85{,}2°$ **OR**	✓ 26,57° ✓ 111,80° ✓answer (3)

	$\tan \hat{CDO} = \frac{1}{2}$ $\hat{CDO} = 26,56505....°$ $\tan(180° - \alpha) = \frac{5}{2}$ $180° - \alpha = 68,19859051...°$ $\hat{DCB} = 180° - (26,56505....° + 68,19859051...°)$ $= 85,236359°$ $= 85,24°$	✓26,57° ✓ 68,2° ✓answer (3)
	OR $\hat{DCB} = \alpha - \hat{CDO}$ $\tan \hat{DCB} = \frac{m_{CB} - m_{CD}}{1 + m_{CB}.m_{CD}}$ $= \frac{-\frac{5}{2} - \frac{1}{2}}{1 + \left(-\frac{5}{2}\right)\left(\frac{1}{2}\right)}$ $= 12$ $\hat{DCB} = 85,24°$	✓ $\tan \hat{DCB} = \frac{m_{CB} - m_{CD}}{1 + m_{CB}.m_{CD}}$ ✓ substitution ✓ answer (3)
	OR $AC = \sqrt{45} \quad BC = \sqrt{116} \quad AB = \sqrt{173}$ $\cos \hat{ACB} = \frac{AC^2 + BC^2 - AB^2}{2AC.BC}$ $= \frac{45 + 116 - 173}{2(\sqrt{45})(\sqrt{116})}$ $= -0,083045...$ $\hat{ACB} = 94,76...°$ $\hat{DCB} = 180° - 94,76...°$ $= 85,24°$	✓ cosine rule ✓ substitution into cosine rule ✓ answer (3)
	OR D(– 11 ; 0) $DC = \sqrt{80} \quad BC = \sqrt{116} \quad DB = \sqrt{180}$ $\cos \hat{DCB} = \frac{DC^2 + BC^2 - DB^2}{2DC.BC}$ $= \frac{80 + 116 - 180}{2(\sqrt{80})(\sqrt{116})}$ $= 0,08304547985...$ $\hat{DCB} = 85,24°$	✓ cosine formula ✓ substitution into cosine rule ✓ answer (3)
	OR	

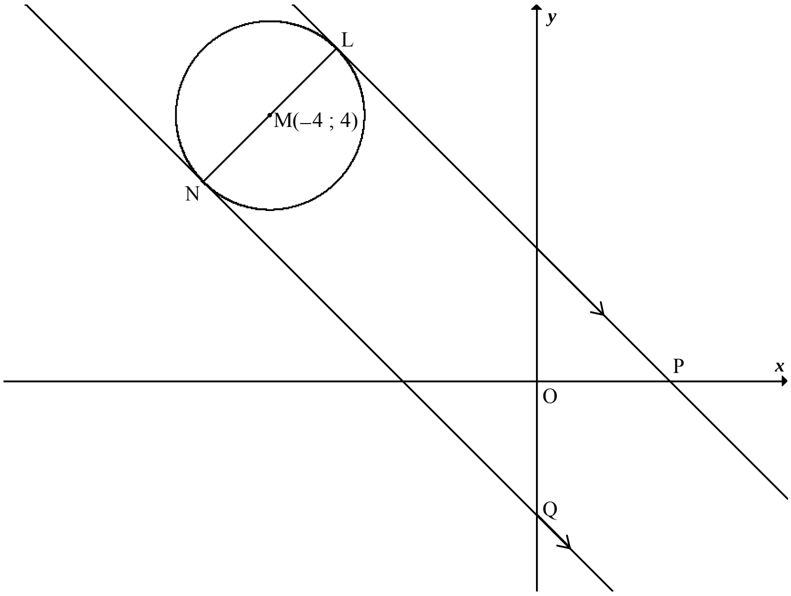
	Equation AC: $2y = x + 11$ D(– 11 ; 0) C(– 3 ; 4) $\text{DC}^2 = (x_C - x_D)^2 + (y_C - y_D)^2$ $= (-3+11)^2 + (4-0)^2$ $= 80$ Equation BC: $2y = -5x - 7$ $\text{P}\left(-\frac{7}{5};0\right)$ $\text{PC}^2 = (-3+\frac{7}{5})^2 + (4-0)^2$ $= \dfrac{464}{25}$ $\text{DP}^2 = (-\frac{7}{5}+11)^2$ $= \dfrac{2304}{25}$ In ΔDCP: $\text{DP}^2 = \text{DC}^2 + \text{CP}^2 - 2\text{DC.CP}.\cos \text{D}\hat{\text{C}}\text{P}$ $\dfrac{2304}{25} = \dfrac{2000}{25} + \dfrac{464}{25} - 2\left(\dfrac{\sqrt{2000}}{5}\right)\left(\dfrac{\sqrt{464}}{5}\right).\cos \text{D}\hat{\text{C}}\text{P}$ $\text{D}\hat{\text{C}}\text{P} = 85{,}23635...$ $\text{D}\hat{\text{C}}\text{P} = 85{,}24°$	✓ cosine formula ✓ substitution into cosine rule ✓ answer (3)

5.3	$y-7=\frac{1}{2}(x-3)$ $y=\frac{1}{2}x+\frac{11}{2}$ $x-2y+11=0$ **OR** $y-4=\frac{1}{2}(x+3)$ $y=\frac{1}{2}x+\frac{11}{2}$ $x-2y+11=0$ **OR** $y=\frac{1}{2}x+c$ $(7)=\frac{1}{2}(3)+c$ $c=\frac{11}{2}$ $y=\frac{1}{2}x+\frac{11}{2}$ $x-2y+11=0$ **Note:** If candidate leaves answer as $y-7=\frac{1}{2}(x-3)$ or $y-4=\frac{1}{2}(x+3)$: **max 1 / 3 marks**	✓ substitution of (3 ; 7) into $y-y_1=m(x-x_1)$ ✓ answer in any form (2) ✓ substitution of (– 3 ; 4) into $y-y_1=m(x-x_1)$ ✓ answer in any form (2) ✓ substitution of (3 ; 7) into $y=mx+c$ ✓ answer in any form (2)
5.4	$\mathrm{M}(x;y)=\left(\frac{-3+1}{2};\frac{4-6}{2}\right)$ $\mathrm{M}(x;y)=(-1;-1)$	✓substitution ✓answer (2)

5.5	$m_{AM} = \frac{7-(-1)}{3-(-1)} = 2$ $y = 2x + c$ $-1 = 2(-1) + c$ $\therefore c = 1$ $y = 2x + 1$ G(a ; b) lies on the line $\therefore b = 2a + 1$ **Note:** If the candidate does not conclude $b = 2a + 1$ from $y = 2x + 1$: **max 3 / 4 marks**	✓gradient = 2 ✓substitution (– 1 ; – 1) ✓$c = 1$ ✓conclusion (4)
	OR $\frac{7-b}{3-a} = \frac{b+1}{a+1}$ $(7-b)(a+1) = (b+1)(3-a)$ $7a + 7 - ab - b = 3b - ab + 3 - a$ $8a - 4b = -4$ $2a - b = -1$ $b = 2a + 1$	✓ $\frac{7-b}{3-a}$ ✓ $\frac{b+1}{a+1}$ ✓equating ✓simplification leading to $2a - b = -1$ (4)
	OR Using the point (– 1 ; – 1) $\frac{b+1}{a+1} = \frac{8}{4}$ $\frac{b+1}{a+1} = 2$ $b + 1 = 2a + 2$ $b = 2a + 1$	✓ substitution of (– 1 ; – 1) into gradient ✓ gradient = 2 ✓ equating ✓ simplification leading to $b + 1 = 2a + 2$ (4)
	OR Using the point (3 ; 7) $\frac{7-b}{3-a} = \frac{8}{4}$ $\frac{7-b}{3-a} = 2$ $7 - b = 6 - 2a$ $b = 2a + 1$	✓ substitution of (3 ; 7) into gradient ✓ gradient = 2 ✓ equating ✓ simplification leading to $7 - b = 6 - 2a$ (4)

5.6	$GC = \sqrt{17}$ $GC^2 = 17$ $(a+3)^2 + (b-4)^2 = 17$ $(a+3)^2 + (2a+1-4)^2 = 17$ $a^2 + 6a + 9 + 4a^2 - 12a + 9 - 17 = 0$ $5a^2 - 6a + 1 = 0$ $(5a-1)(a-1) = 0$ $a = \frac{1}{5}$ or $a = 1$ $\therefore b = \frac{7}{5}$ or $b = 3$ **Note:** If candidate swops a and b around: **max 2 / 6 marks**	✓ distance formula in terms of a and b ✓substitution of $b = 2a + 1$ ✓standard form ✓factors or correct substitution into formula ✓values of a ✓values of b (6)
	OR $a = \frac{b-1}{2}$ $17 = (a+3)^2 + (b-4)^2$ $17 = \left(\left(\frac{b-1}{2}\right) + 3\right)^2 + (b-4)^2$ $17 = \left(\frac{b+5}{2}\right)^2 + (b-4)^2$ $17 = \frac{b^2 + 10b + 25 + 4b^2 - 32b + 64}{4}$ $68 = 5b^2 - 22b + 89$ $0 = 5b^2 - 22b + 21$ $0 = (5b-7)(b-3)$ $\therefore b = \frac{7}{5}$ or $b = 3$	✓ $a = \frac{b-1}{2}$ ✓ distance formula in terms of a and b ✓ substitution of $a = \frac{b-1}{2}$ ✓ standard form ✓ factors or correct substitution into formula ✓ values of b (6) **[20]**

QUESTION 6

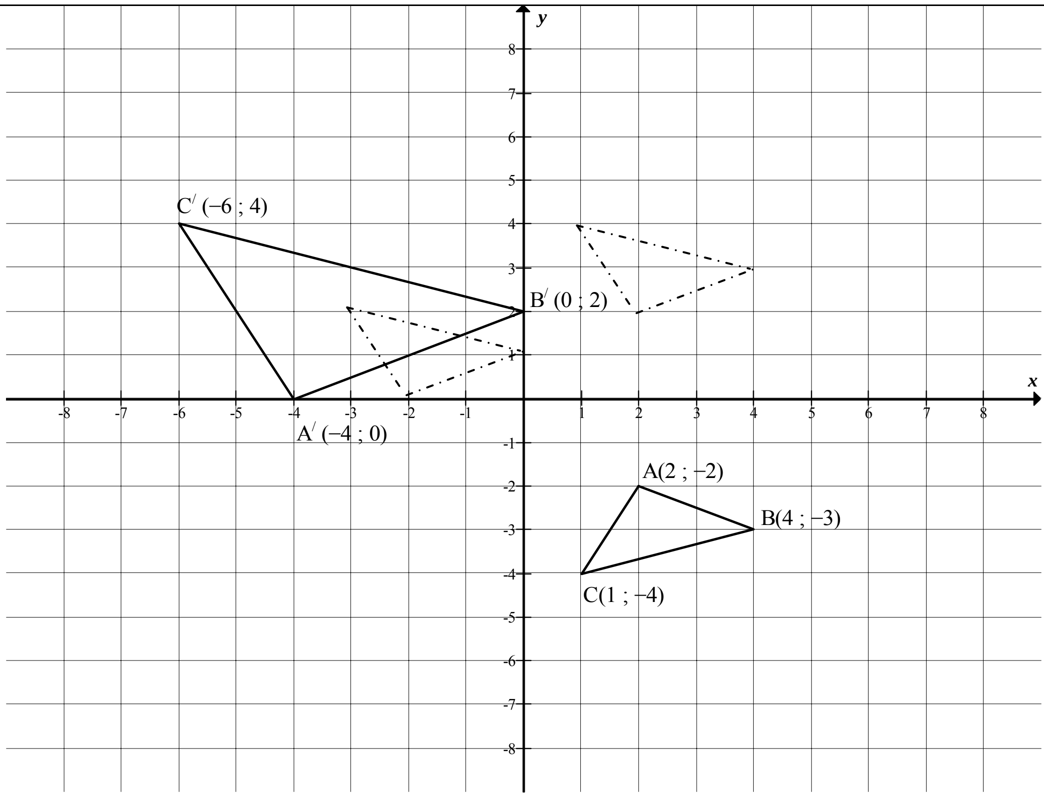


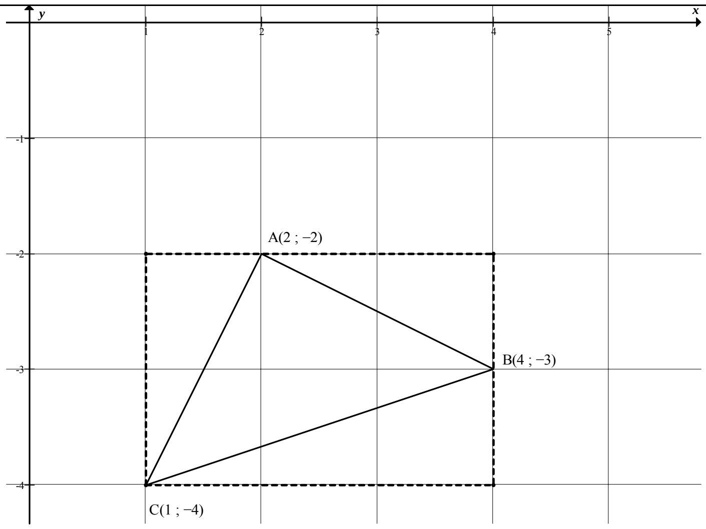
6.1	$y=-x+2$ $m_{LP}=-1$ $\therefore m_{LN}=\frac{-1}{-1}=1$ $y=x+c$ $4=-4+c$ $\therefore c=8$ $y=x+8$ **Note:** If candidate leaves it as $y-4=x+4$ **max 2 / 3 marks** Answer only: **Full marks** **OR** $y-4=1(x+4)$ $y=x+8$	✓ $m_{LP}=-1$ ✓ $m_{LN}=1$ ✓equation (3) ✓ $m=1$ ✓ substitution of $y-y_1=m(x-x_1)$ ✓ answer (3)
6.2	$x+8=-x+2$ $2x=-6$ $x=-3$ $y=-3+8$ $y=5$ L$(-3;5)$ **OR** $y+x=2$...........(1) $y-x=8$...........(2 $2y=10$ $\therefore y=5$ $\therefore x=-3$ L$(-3;5)$ **Note:** No penalty for not leaving in coordinate form	✓x-value ✓y-value Equations leading to these values must be used (2)

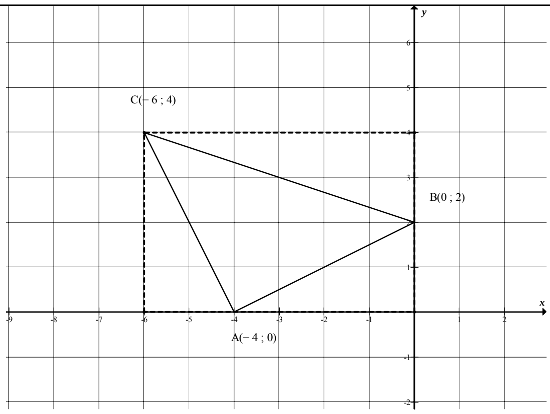
6.3	$(x+4)^2+(y-4)^2=r^2$ $(-3+4)^2+(5-4)^2=r^2$ $\therefore r^2=2$ $(x+4)^2+(y-4)^2=2$ Equation can be left as: $x^2+8x+y^2-8y+30=0$ **Note:** If the candidate only uses the distance formula to determine the radius $(-3+4)^2+(5-4)^2=r^2$ $\therefore r^2=2$ then **2 / 3 marks**	✓ $(x+4)^2+(y-4)^2=r^2$ ✓substitution of (– 3 ; 5) ✓ $r^2=2$ (3)
6.4	Let N(x, y). Since M(– 4 ; 4) is the midpoint of LN and L(– 3 ; 5) $\frac{x-3}{2}=-4$; $\frac{y+5}{2}=4$ $\therefore x=-5$; $y=3$ **OR** $y=x+8$ $(x+4)^2+(y-6)^2=2$ $(x+4)^2+(x+8-4)^2-2=0$ $x^2+8x+16+x^2+8x+16-2=0$ $2x^2+16x+30=0$ $x^2+8x+15=0$ $(x+5)(x+3)=0$ $x=-3$ or $x=-5$ $y=5$ $y=3$ $\therefore$ N(– 5 ; 3) **Note:** Answer only: **Full marks**	✓ using the fact that M is the midpoint of LN ✓ $x=-5$ ✓ $y=3$ (3) ✓ $(x+4)^2+(x+8-4)^2-2=0$ ✓ $x=-5$ ✓ $y=3$ (3)
6.5	$m_{NQ}=-1$ $y=-x+c$ $3=-(-5)+c$ $c=-2$ $y=-x-2$ **Note:** Answer only: **Full marks** **OR** $m_{NQ}=-1$ $y-3=-(x+5)$ $y=-x-2$ **OR** Equation of LP is $x+y=2$ NQ || LP $\therefore$ equation of NQ is $x+y=k$ for some $k\in R$ But N(– 5 ; 3) lies on NQ $\therefore x+y=-5+3=-2$	✓gradient ✓substitution of (– 5 ; 3) into $y = mx + c$ ✓$c=-2$ (3) ✓gradient ✓substitution of (– 5 ; 3) into $y-y_1=m(x-x_1)$ ✓equation (3) ✓ $x+y=k$ ✓ substitution of (– 5 ; 3) ✓ equation (3)

	OR NQ is a reflection of LP ($y + x = 2$) in the line $y = x$ $\therefore$ equation of NQ is $x + y = -2$	
6.6	Let new radius of circle be R and centre be $M^{/}$. $M'(-4+6\,;4)$ $=(2\,;4)$ $R = 2r$ $R^2 = 4r^2$ $= 4(2)$ $= 8$ $\therefore (x-2)^2 + (y-4)^2 = (2\sqrt{2})^2$ $\therefore (x-2)^2 + (y-4)^2 = 8$ **OR** Let R = new radius of circle $R^2 = (2r)^2 = 4(2) = 8$ $(x-6+4)^2 + (y-4)^2 = 8$ $\therefore (x-2)^2 + (y-4)^2 = 8$	✓ $M'(2\,;4)$ ✓ $r = 2\sqrt{2}$ ✓ equation (3) ✓ $(x-2)^2$ ✓ $(y-4)^2$ ✓8 or $\left(2\sqrt{2}\right)^2$ (3) **[17]**

QUESTION 7

7.1	 **Note:** • If the candidate only draws the correct triangle with labels, **full marks** • If they plot the points correctly and do not draw the triangle, **max 5 / 6 marks** • In the 3 sketches, if one vertex of the three is wrong, then 1 / 2 marks for the incorrect sketch, then CA applies. • If they write down the points and do not plot the points and draw the triangle **max 3 / 6 marks** • If the vertices are correct but not labelled and the points are joined **max 5 / 6 marks** • If the vertices are correct, not labelled and not joined **max 4 / 6 marks** • If a candidate finds a formula first and gets it wrong Max 1 mark for the formula Max 2 marks for the calculation of $A^/$, $B^/$, $C^/$ coordinates (CA) 1 mark for plotting 3 vertices 1 mark for completing the triangle and labelling	2 marks for each diagram of the transformation (6) **OR** If candidate works out the general rule first $(x ; y) \rightarrow (2x - 8; -2y - 4)$ ✓ $A^/$ (–4 ; 0) ✓ $B^/$ (0 ; 2) ✓ $C^/$ (–6 ; 4) 1 mark per each correct point plotted and joined (6)

7.2	$(x\,;y)\rightarrow(x\,;-y)$ $(x\,;-y)\rightarrow(x-4\,;-y-2)$ $(x-4\,;-y-2)\rightarrow(2x-8\,;-2y-4)$ **Note:** Answer only: Full marks **OR** $(x\,;y)\rightarrow(2x-8;-2y-4)$ **Note:** • If the candidate writes $(x\,;y)\rightarrow(x\,;-y)$ $(x\,;y)\rightarrow(x-4;y-2)$ $(x\,;y)\rightarrow(2x\,;2y)$ then **2 / 4 marks** • If the candidate writes $(x\,;y)\rightarrow(x\,;-y)$ $(x\,;y)\rightarrow(x-4;-y-2)$ $(x\,;y)\rightarrow(2x-8\,;-2y-4)$ then **4 / 4 marks** • If candidate writes $(x\,;y)\rightarrow 2(x-4;-y-2)$ then **3 / 4 marks**	✓ $(x\,;y)\rightarrow(x\,;-y)$ ✓ $x-4$ ✓ $-y-2$ ✓ $(2x-8\,;-2y-4)$ (4)
7.3	 Area Δ ABC = area of rectangle – sum of 3 triangle areas $= 6-\left(1+\frac{3}{2}+1\right)$ $=\frac{5}{2}$ Area $\Delta A'B'C' = 2^2\left(\frac{5}{2}\right)$ $=10\text{ units}^2$ **OR**	✓ $6-\left(1+\frac{3}{2}+1\right)$ ✓ $\frac{5}{2}$ ✓✓ 10 (4)



$$\text{Area } \Delta A'B'C' = 24 - \left(\left(\frac{1}{2}.6.2\right) + \frac{1}{2}(4)(2) + \frac{1}{2}(2)(4)\right)$$
$$= 24 - 6 - 4 - 4$$
$$= 10 \text{ units}^2$$

✓ $\left(\left(\frac{1}{2}.6.2\right) + \frac{1}{2}(4)(2) + \frac{1}{2}(2)(4)\right)$
✓ 24
✓✓ 10
(4)

OR

$m_{AC} = 2$ and $m_{AB} = -\frac{1}{2}$ $\therefore$ product = – 1
$\therefore \hat{CAB} = 90°$
$AB = \sqrt{5}$
$AC = \sqrt{5}$
$\therefore$ Area ΔABC is $\frac{1}{2}\left(\sqrt{5}\right)^2 = \frac{5}{2}$
$\therefore$ Area Δ $A'B'C'$ is $4 \times \frac{5}{2} = 10$ square units

✓ $AB = \sqrt{5}$ and $AC = \sqrt{5}$
✓ $\frac{5}{2}$
✓✓ 10
(4)

OR

$m_{A'C'} = -2$ and $m_{A'B'} = \frac{1}{2}$ $\therefore$ product = – 1
$\therefore C'\hat{A}'B' = 90°$
$A'B' = \sqrt{20}$
$A'C' = \sqrt{20}$
$\therefore$ Area $\Delta A'B'C'$ is $\frac{1}{2}\left(\sqrt{20}\right)^2 = 10$ square units

✓ $A'B' = \sqrt{20}$
✓ $A'C' = \sqrt{20}$
✓✓ 10
(4)

OR

$AB = \sqrt{5}$
$AC = \sqrt{5}$
$BC^2 = 10$
$BC = \sqrt{10}$
$\perp$height = $\sqrt{\frac{10}{4}}$
Area of $\Delta ABC = \frac{1}{2}.\sqrt{\frac{10}{4}}.\sqrt{10} = \frac{5}{2}$
Area $\Delta A'B'C' = 4 \times \frac{5}{2} = 10$ square units

✓ $AB = \sqrt{5}$ and $AC = \sqrt{5}$
✓ $\frac{5}{2}$
✓✓ answer
(4)

✓ $AB = \sqrt{5}$ and $AC = \sqrt{5}$

OR

$AB = \sqrt{5}$

$AC = \sqrt{5}$

$BC^2 = 10$

$BC = \sqrt{10}$

$AC^2 + AB^2 = BC^2$

$\therefore C\hat{A}B = 90°$

Area $\Delta ABC = \; = \frac{1}{2}.\left(\sqrt{5}\right)^2 = \frac{5}{2}$

Area $\Delta A^{/}B^{/}C^{/} = 4 \times \frac{5}{2} = 10$ square units

✓ $\frac{5}{2}$
✓✓ answer
(4)

OR

Area $\Delta ABC \quad = \frac{1}{2}.bc \sin A$

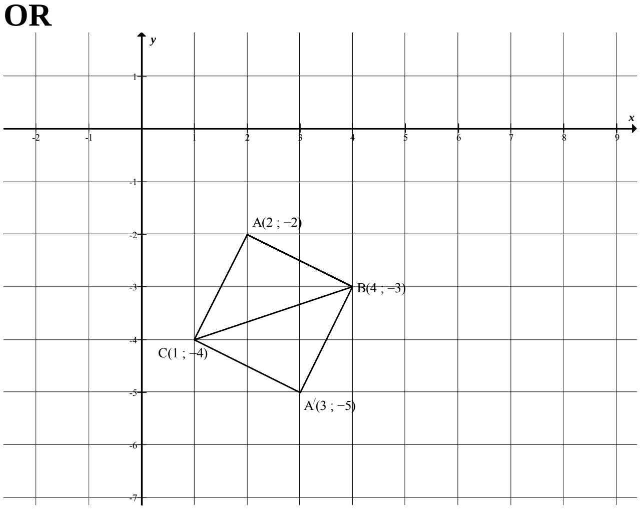
$= \frac{1}{2}\left(\sqrt{5}\right)\left(\sqrt{5}\right)\sin 90$

$= \frac{5}{2}$

Area $\Delta A^{/}B^{/}C^{/} = 4 \times \frac{5}{2} = 10$ square units

✓ $AB = \sqrt{5}$ and $AC = \sqrt{5}$
✓ $\frac{5}{2}$
✓✓ 10
(4)

OR



Reflect ΔABC about CB and get the square $ABA^{//}C$ of side $\sqrt{5}$

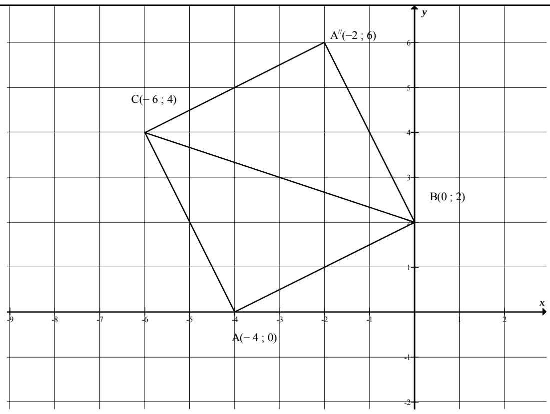
Area square $= \left(\sqrt{5}\right)\left(\sqrt{5}\right) = 5$

Area $\Delta ABC = \frac{5}{2}$

Area $\Delta A^{/}B^{/}C^{/} = 4 \times \frac{5}{2} = 10$ square units

✓ $AB = \sqrt{5}$ and $AC = \sqrt{5}$
✓ reflection to get square
✓✓ 10
(4)

OR

	 Reflect $\Delta A^/B^/C^/$ about $C^/B^/$ and get the square $A^/B^/A^{//}C^/$ of side $\sqrt{20}$ Area square $=\left(\sqrt{20}\right)\left(\sqrt{20}\right)=20$ Area $\Delta A^/B^/C^/ = \frac{1}{2}\times 20 = 10$ square units	✓ $A^/B^/=\sqrt{20}$ and $A^/C^/=\sqrt{20}$ ✓ reflection to get square ✓✓ 10 (4) **[14]**

QUESTION 8

<table>
<tr><td>8.1</td><td>

$x' = x\cos\alpha - y\sin\alpha$
$= 2\cos 75° - 4\sin 75°$
$= 2\cos(30° + 45°) - 4\sin(30° + 45°)$
$= 2\cos 30°\cos 45° - 2\sin 30°\sin 45° - 4\sin 30°\cos 45° - 4\cos 30°\sin 45°$
$= 2.\frac{\sqrt{3}}{2}.\frac{\sqrt{2}}{2} - 2.\frac{1}{2}.\frac{\sqrt{2}}{2} - 4.\frac{1}{2}.\frac{\sqrt{2}}{2} - 4\frac{\sqrt{3}}{2}.\frac{\sqrt{2}}{2}$
$= \frac{2\sqrt{6} - 2\sqrt{2} - 4\sqrt{2} - 4\sqrt{6}}{4}$
$= \frac{-2\sqrt{2} - 6\sqrt{6}}{4}$
$= \frac{-\sqrt{6} - 3\sqrt{2}}{2}$ or $-\frac{1}{\sqrt{2}}(3 + \sqrt{3})$

Note:
If the candidate uses a calculator i.e. gives a decimal answer
max 5 / 6 marks

Incorrect formula:
max 5 / 6 marks

OR

$x' = x\cos\alpha + y\sin\alpha$
$= 2\cos(-75°) + 4\sin(-75°)$
$= 2\cos(75°) - 4\sin(75°)$
$= 2\cos(30° + 45°) - 4\sin(30° + 45°)$
$= 2\cos 30°\cos 45° - 2\sin 30°\sin 45° - 4\sin 30°\cos 45° - 4\cos 30°\sin 45°$
$= 2.\frac{\sqrt{3}}{2}.\frac{\sqrt{2}}{2} - 2.\frac{1}{2}.\frac{\sqrt{2}}{2} - 4.\frac{1}{2}.\frac{\sqrt{2}}{2} - 4\frac{\sqrt{3}}{2}.\frac{\sqrt{2}}{2}$
$= \frac{2\sqrt{6} - 2\sqrt{2} - 4\sqrt{2} - 4\sqrt{6}}{4}$
$= \frac{-2\sqrt{2} - 6\sqrt{6}}{4}$
$= \frac{-\sqrt{6} - 3\sqrt{2}}{2}$ or $-\frac{1}{\sqrt{2}}(3 + \sqrt{3})$

OR

</td><td>

✓substitution using anti clockwise formula

✓ 75 = 30 + 45

✓ cos expansion
✓ sin expansion
✓ substitution of special angles

✓simplified answer for x'

(6)

✓substitution using clockwise formula

✓ 75 = 30 + 45

✓ cos expansion
✓ sin expansion
✓ substitution of special angles

✓simplified answer for x'

(6)

</td></tr>
</table>

	First compute $\cos 75° = \cos(30° + 45°)$ $= \cos 30°.\cos 45° - \sin 30°.\sin 45°$ $= \frac{\sqrt{3}}{2}.\frac{\sqrt{2}}{2} - \frac{1}{2}.\frac{\sqrt{2}}{2}$ $= \frac{\sqrt{6} - \sqrt{2}}{4}$ and $\sin 75° = \sin(30° + 45°)$ $= \sin 30°.\cos 45° + \cos 30°.\sin 45°$ $= \frac{1}{2}.\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}.\frac{\sqrt{2}}{2}$ $= \frac{\sqrt{2} + \sqrt{6}}{4}$ $x^{/} = 2\cos 75° - 4\sin 75°$ $= 2\left(\frac{\sqrt{6} - \sqrt{2}}{4}\right) - 4\left(\frac{\sqrt{2} + \sqrt{6}}{4}\right)$ $= \frac{-2\sqrt{6} - 6\sqrt{2})}{4}$ $= \frac{-\sqrt{6} - 3\sqrt{2}}{2}$	✓ 75 = 30 + 45 ✓ cos expansion ✓ substitution of special angles in the first expansion ✓ sin expansion ✓ substitution ✓simplified answer for $x^{/}$ (6)

8.2	$x^{/} = x\cos\beta - y\sin\beta$ $3\cos\beta - \sin\beta = \dfrac{3-\sqrt{3}}{2}$ … (1) $y^{/} = y\cos\beta + x\sin\beta$ $\cos\beta + 3\sin\beta = \dfrac{1+3\sqrt{3}}{2}$ … (1*) (1)×3 + (1)*: $10\cos\beta = 3\left(\dfrac{3-\sqrt{3}}{2}\right) + \dfrac{(1+3\sqrt{3}}{2}$ $= \dfrac{1}{2}\left(9-3\sqrt{3}+1+3\sqrt{3}\right)$ $= 10\left(\dfrac{1}{2}\right)$ $\therefore \cos\beta = \dfrac{1}{2}$ $\therefore \beta = 60°$ **OR** (1) – 3(1)*: $-10\sin\beta = 3\left(\dfrac{1}{2}\right) - \left(\dfrac{3-\sqrt{3}}{2}\right)$ $= \dfrac{3-\sqrt{3}-3(1+3\sqrt{3})}{2}$ $= \dfrac{-10\sqrt{3}}{2}$ $\therefore \sin\beta = \dfrac{\sqrt{3}}{2}$ $\therefore \beta = 60°$	✓ substitution into $x^{/}$ ✓ substitution into $y^{/}$ ✓ simplification ✓ solving simultaneously ✓ $\dfrac{\sqrt{3}}{2}$ or $\dfrac{1}{2}$ ✓answer (6)
	OR $3\cos\beta - \sin\beta = \dfrac{3-\sqrt{3}}{2}$ … (1) $\cos\beta = \dfrac{1+3\sqrt{3}}{2} - 3\sin\beta$ … (2) Substitute (2) into (1) $3\left(\dfrac{1+3\sqrt{3}}{2} - 3\sin\beta\right) - \sin\beta = \dfrac{3-\sqrt{3}}{2}$ $\dfrac{3+9\sqrt{3}}{2} - 9\sin\beta - \sin\beta = \dfrac{3-\sqrt{3}}{2}$ $-10\sin\beta = \dfrac{3-\sqrt{3}-3-9\sqrt{3}}{2}$ $-10\sin\beta = \dfrac{-10\sqrt{3}}{2}$ $\sin\beta = \dfrac{\sqrt{3}}{2}$ $\beta = 60°$	✓ equation (1) ✓ equation (2) ✓substitution ✓simplification ✓ $\sin\beta = \dfrac{\sqrt{3}}{2}$ ✓answer (6)

OR

$$3\cos\beta - \sin\beta = \frac{3-\sqrt{3}}{2}$$

and $\cos\beta + 3\sin\beta = \frac{1+3\sqrt{3}}{2}$

Try $\beta = 60°$

$$3\cos\beta - \sin\beta = 3\left(\frac{1}{2}\right) - 3\left(\frac{\sqrt{3}}{2}\right) = \frac{3-\sqrt{3}}{3}$$

$$\cos\beta + 3\sin\beta = \frac{1}{2} + 3\left(\frac{\sqrt{3}}{2}\right) = \frac{1+3\sqrt{3}}{2}$$

$\therefore \beta = 60°$

Note:
Answer only:
max 2 / 6 marks

✓✓ $\beta = 60°$

✓ substitution
✓ simplification

✓ substitution
✓ simplification

(6)

OR

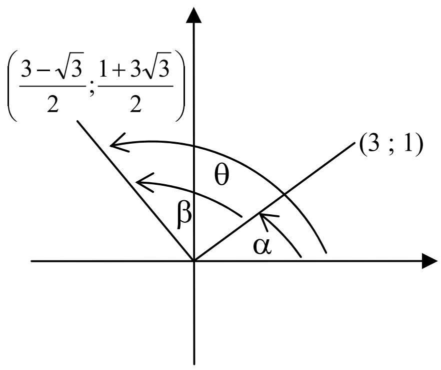
$$\tan\alpha = \frac{1}{3}$$

$$\alpha = 18{,}43°$$

$$\tan\theta = \frac{\frac{1+3\sqrt{3}}{2}}{\frac{3-\sqrt{3}}{2}} = \frac{1+3\sqrt{3}}{3-\sqrt{3}}$$

$$\theta = 78{,}43°$$

$$\beta = 78{,}43° - 18{,}43° = 60°$$



$$\left(\frac{3-\sqrt{3}}{2};\frac{1+3\sqrt{3}}{2}\right)$$

✓ $\tan\alpha = \frac{1}{3}$
✓ $\alpha = 18{,}43°$

✓ $\tan\theta = \frac{1+3\sqrt{3}}{3-\sqrt{3}}$

✓ $\theta = 78{,}43°$

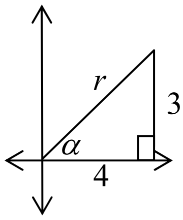
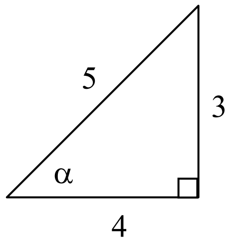
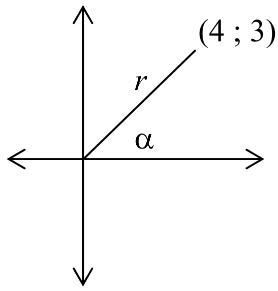
✓ simplification
✓answer

(6)

OR

	$x^{/} = x\cos\beta - y\sin\beta$ $\frac{3-\sqrt{3}}{2}\cos\beta - \frac{1+3\sqrt{3}}{2}\sin\beta = 3$ $3\cos\beta - \sqrt{3}\cos\beta - \sin\beta - 3\sqrt{3}\sin\beta = 6$ $\frac{1+3\sqrt{3}}{2}\cos\beta + \frac{3-\sqrt{3}}{2}\sin\beta = 1$ $\cos\beta + 3\sqrt{3}\cos\beta + 3\sin\beta - \sqrt{3}\sin\beta = 2$ $3\cos\beta + 9\sqrt{3}\cos\beta + 9\sin\beta - 3\sqrt{3}\sin\beta = 6$ $3\cos\beta - \sqrt{3}\cos\beta - \sin\beta - 3\sqrt{3}\sin\beta = 6$ $10\sqrt{3}\cos\beta + 10\sin\beta = 0$ $\sin\beta = -\sqrt{3}\cos\beta$ $\cos\beta + 3\sqrt{3}\cos\beta + 3(-\sqrt{3}\cos\beta) - \sqrt{3}(-\sqrt{3}\cos\beta) = 2$ $4\cos\beta = 2$ $\cos\beta = \frac{1}{2}$ $\beta = 60°$	✓ substitution ✓substitution ✓simplification ✓ $\sin\beta = -\sqrt{3}\cos\beta$ ✓ $\cos\beta = \frac{1}{2}$ ✓answer (6) **[12]**

QUESTION 9

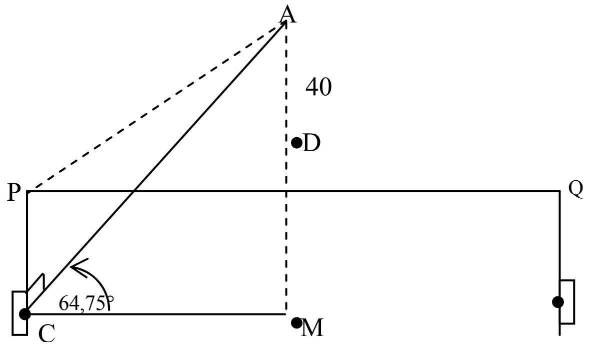
9.1	   $r=\sqrt{16+9}=5$ (Pyth) $\sin\alpha=\frac{3}{5}$ Accept : 0,6	✓ any one of the diagram ✓value of r ✓answer (3)
9.2	$\cos^2(90°-\alpha)-1$ $=\sin^2\alpha-1$ $=\left(\frac{3}{5}\right)^2-\frac{25}{25}$ $=\frac{-16}{25}$ $=-0{,}64$ **OR** $\cos^2(90°-\alpha)-1$ $=\sin^2\alpha-(\sin^2\alpha+\cos^2\alpha)$ $=-\cos^2\alpha$ $=-\left(\frac{4}{5}\right)^2$ $=\frac{-16}{25}$ $=-0{,}64$	✓ $\cos(90°-\alpha)=\sin\alpha$ ✓ substitution of $\sin\alpha=\frac{3}{5}$ (2)
9.3	$1-\sin 2\alpha$ $=1-2\sin\alpha\cos\alpha$ $=1-2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$ $=1-\frac{24}{25}$ $=\frac{1}{25}$ **OR** $1-\sin 2\alpha$ $=\sin^2\alpha-2\sin\alpha\cos\alpha+\cos^2\alpha$ $=(\sin\alpha-\cos\alpha)^2$ $=\left(\left(\frac{3}{5}\right)-\left(\frac{4}{5}\right)\right)^2$ $=\left(-\frac{1}{5}\right)^2$ $=\frac{1}{25}$	✓$\sin 2\alpha=2\sin\alpha.\cos\alpha$ ✓ $\frac{24}{25}$ ✓answer (3) ✓ $\sin^2\alpha+\cos^2\alpha=1$ ✓ $\left(\left(\frac{3}{5}\right)-\left(\frac{4}{5}\right)\right)^2$ ✓answer (3) **[8]**

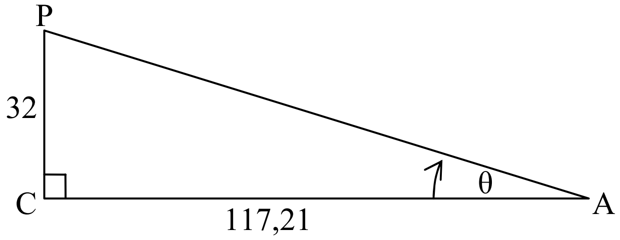
QUESTION 10

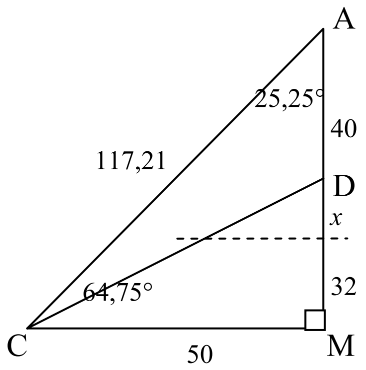
10.1	$\dfrac{\sin(90°+\theta)+\cos(180°+\theta)\sin(-\theta)}{\sin 180°-\tan 135°}$ $=\dfrac{\cos\theta+(-\cos\theta)(-\sin\theta)}{0+1}$ $=\cos\theta+\cos\theta.\sin\theta$ $=\cos\theta(1+\sin\theta)$	✓ $\cos\theta$ ✓ $-\cos\theta$ ✓ $-\sin\theta$ ✓ $0+1$ ✓answer (5)
10.2	$\dfrac{4\sin A\cos A\cos 2A.\sin 15°}{\sin 2A(1-2\sin^2 A)}$ $=\dfrac{4\sin A\cos A\cos 2A.\sin 15°}{2\sin A\cos A(1-2\sin^2 A)}$ $=\dfrac{2\cos 2A.\sin 15°}{\cos 2A}$ $=2\sin 15°$ $=2\sin(45°-30°)$ $=2[\sin 45°\cos 30°-\cos 45°\sin 30°]$ $=2\left[\dfrac{\sqrt{2}}{2}.\dfrac{\sqrt{3}}{2}-\dfrac{\sqrt{2}}{2}.\dfrac{1}{2}\right]$ $=2\left[\dfrac{\sqrt{6}}{4}-\dfrac{\sqrt{2}}{4}\right]$ $=\dfrac{\sqrt{6}-\sqrt{2}}{2}$	✓ $2\sin A\cos A$ ✓ $1-2\sin^2 A=\cos 2A$ ✓ $2\sin 15°$ ✓ $15=45-30$ or $15=60-45$ ✓substitution ✓ $2\left[\dfrac{\sqrt{6}}{4}-\dfrac{\sqrt{2}}{4}\right]$ (6)
	OR Left Hand Side $=\dfrac{4\sin A\cos A\cos 2A.\sin 15°}{2\sin A\cos A(1-2\sin^2 A)}$ $=\dfrac{2\cos 2A.\sin 15°}{\cos 2A}$ $=2\sin 15°$ $=2\sin(60°-45°)$ $=2[\sin 60°\cos 45°-\cos 60°\sin 45°]$ $=2\left[\dfrac{\sqrt{3}}{2}.\dfrac{\sqrt{2}}{2}-\dfrac{1}{2}.\dfrac{\sqrt{2}}{2}\right]$ $=2\left[\dfrac{\sqrt{6}}{4}-\dfrac{\sqrt{2}}{4}\right]$ $=\dfrac{\sqrt{6}-\sqrt{2}}{2}=RHS$	✓ $2\sin A\cos A$ ✓ $1-2\sin^2 A=\cos 2A$ ✓ $2\sin 15°$ ✓ $15=45-30$ or $15=60-45$ ✓substitution ✓ $2\left[\dfrac{\sqrt{6}}{4}-\dfrac{\sqrt{2}}{4}\right]$ (6)

	OR $\text{Left Hand Side} = \dfrac{4\sin A\cos A\cos 2A.\sin 15°}{2\sin A\cos A(1-2\sin^2 A)}$ $= \dfrac{2\sin 2A\cos 2A.\sin 15°}{\sin 2A\cos 2A}$ $= 2\sin 15°$ $= 2\sin(45°-30°)$ $= 2[\sin 45°\cos 30° - \cos 45°\sin 30°]$ $= 2\left[\dfrac{\sqrt{2}}{2}.\dfrac{\sqrt{3}}{2} - \dfrac{\sqrt{2}}{2}.\dfrac{1}{2}\right]$ $= 2\left[\dfrac{\sqrt{6}}{4} - \dfrac{\sqrt{2}}{4}\right]$ $= \dfrac{\sqrt{6}-\sqrt{2}}{2} = RHS$	
10.3	$6\cos x - 5 = \dfrac{4}{\cos x}$ $6\cos^2 x - 5\cos x = 4$ $6\cos^2 x - 5\cos x - 4 = 0$ $(3\cos x - 4)(2\cos x + 1) = 0$ $\cos x = \dfrac{4}{3}$ *or* $\cos x = \dfrac{-1}{2}$ no solution or $x = 120° + k.360°\ , k \in Z$ or $x = 240° + k.360°\ , k \in Z$ Alternative solution for $\cos x = \dfrac{-1}{2}$ $x = k.360° \pm 120°\ k \in Z$ **Note:** If candidate puts $\pm k.360$ then $k \in N_0$	✓ standard form ✓ factors ✓ both equations ✓ $240° + k.360°$ ✓ $120° + k.360°$ ✓ $k \in Z$ (6) **[17]**

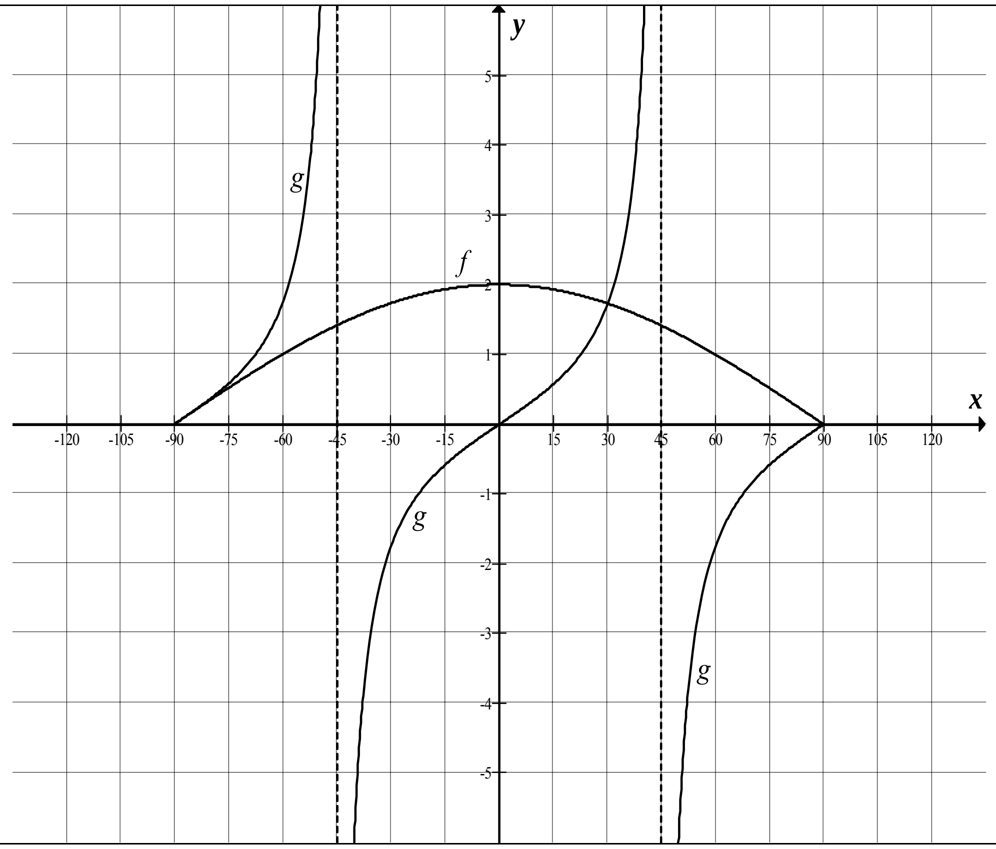
QUESTION 11



11.1	$\cos 64{,}75° = \dfrac{50}{AC}$ $AC = \dfrac{50}{\cos 64{,}75°}$ $= 117{,}21\text{ m}$ **OR** $AC = \dfrac{50}{\cos 64{,}75°}$ $\therefore AC = 117{,}2144026\text{ m}$ $\therefore AC = 117{,}21\text{m}$ **OR** $\dfrac{50}{\sin 25{,}25°} = \dfrac{AC}{\sin 90°}$ $\therefore AC = \dfrac{50\sin 90°}{\sin 25{,}25°}$ $\therefore AC = 117{,}21\text{ m}$	✓ substitution in ratio ✓AC subject of the formula ✓ answer (3) ✓ substitution into ratio ✓AC subject of the formula ✓ answer (3) ✓ substitution into sine rule ✓AC subject of the formula ✓ answer (3)
11.2	PC is given to be $\frac{1}{2}(64) = 32$ m  $\tan P\hat{A}C = \dfrac{32}{117{,}21}$ $\theta = 15{,}27°$ (15,27042173...) **Note:** If the candidate takes the unrounded answer for AC, then the answer is 15,27° (15,26987495...)	✓ $\tan P\hat{A}C$ ✓ $\dfrac{32}{117{,}21}$ ✓ answer (3)

11.3	$CD^2 = 117,21^2 + 40^2 - 2(117,21)(40)\cos 25,25$ $= 6857,289092$ $\therefore CD = 82,81 \quad m$ **Note:** If don't use the rounded off then CD = 82,81 m. Accept this answer. **OR**  AM = ACsin 64,75° **OR** AM = CM tan 64,75° **OR** AM = AC cos 25,25° = 106,0111876 = 50 tan 64,75° = 117,21.cos25,25° = 106,01 = 106,01 = 106,01 DM = 106,01 – 40 = 66,01 $CD^2 = CM^2 + DM^2$ $= (50)^2 + (66,01)^2$ = 6857,3201 CD = 82,81 metres **OR** AM = ACsin 64,75° **OR** AM = CM tan 64,75° **OR** AM = AC cos 25,25° = 106,0111876 = 50 tan 64,75° = 117,21.cos25,25° = 106,01 = 106,01 = 106,01 DM = 106,01 – 40 = 66,01 $DC^2 = (50)^2 + (66,01)^2 - 2(50)(66,01).\cos 90°$ = 6857,3201 CD = 82,81 metres **OR** $\sin 64,75° = \dfrac{40 + x + 32}{117,21}$ $x = 34,01$ $CD^2 = CM^2 + DM^2$ $= (50)^2 + (32 + 34,01)^2$ = 6857,3201 CD = 82,81 metres	✓ cos rule ✓✓substitution ✓answer (4) ✓ AM = 106,01 ✓ DM = 66,01 ✓ Pythagoras ✓answer (4) ✓ AM = 106,01 ✓ DM = 66,01 ✓ cos rule ✓answer (4) **[10]**

QUESTION 12

12.1		tan 2x: ✓asymptotes (45° & –45°) ✓x-intercepts ✓ middle curve ✓end curves **Note:** The curve must tend toward the asymptotes 2 cos x: ✓y-intercept ✓x-intercepts (6) **Note:** If the candidate draws outside of the interval, NO PENALTY.

12.2	For these values of x, $\cos 2x \neq 0$ $2\cos x = \dfrac{\sin 2x}{\cos 2x}$ $= \dfrac{2\sin x\cos x}{1-2\sin^2 x}$ $1-2\sin^2 x = \sin x$ or $\cos x = 0$ $2\sin^2 x + \sin x - 1 = 0$ or $\cos x = 0$ $(\sin x + 1)(2\sin x - 1) = 0$ $\sin x = -1$ or $\sin x = \dfrac{1}{2}$ or $\cos x = 0$ $x = \pm 90°$ *or* $x = 30°$ **OR** For these values of x, $\cos 2x \neq 0$ $2\cos x = \dfrac{\sin 2x}{\cos 2x}$ $= \dfrac{2\sin x\cos x}{2\cos^2 x - 1}$ $2\cos x(2\cos^2 x - 1) = 2\sin x\cos x$ $2\cos x(2(1-\sin^2 x) - 1) = 2\sin x\cos x$ $2\cos x(1-2\sin^2 x) - 2\sin x\cos x = 0$ $2\cos x(2\sin^2 x + \sin x - 1) = 0$ $2\sin^2 x + \sin x - 1 = 0$ or $\cos x = 0$ $(\sin x + 1)(2\sin x - 1) = 0$ $\sin x = -1$ or $\sin x = \dfrac{1}{2}$ or $\cos x = 0$ $x = \pm 90°$ *or* $x = 30°$ **OR** $2\cos x = \dfrac{\sin 2x}{\cos 2x}$ $2\cos x.\cos 2x = \sin 2x$ $2\cos x.\cos 2x - 2\sin x.\cos x = 0$ $2\cos x(\cos 2x - \sin x) = 0$ $\cos 2x = \sin x$ or $2\cos x = 0$ $1-2\sin^2 x = \sin x$ $2\sin^2 x + \sin x - 1 = 0$ $(\sin x + 1)(2\sin x - 1) = 0$ $\sin x = -1$ or $\sin x = \dfrac{1}{2}$ or $\cos x = 0$ $x = \pm 90°$ *or* $x = 30°$	✓ $\dfrac{\sin 2x}{\cos 2x}$ ✓ $2\sin x\cos x$ ✓ $1-2\sin^2 x$ ✓ $\cos x = 0$ ✓ factors ✓ equations ✓ $\pm 90°$ ✓ 30^0 (8) ✓ $\dfrac{\sin 2x}{\cos 2x}$ ✓ $2\sin x\cos x$ ✓ $2\cos^2 x - 1$ ✓ $\cos x = 0$ ✓ factors ✓ equations ✓ $\pm 90°$ ✓ 30^0 (8) ✓ $\dfrac{\sin 2x}{\cos 2x}$ ✓ $2\sin x\cos x$ ✓ $1-2\sin^2 x$ ✓ $\cos x = 0$ ✓ factors ✓ equations ✓ $\pm 90°$ ✓ 30^0 (8)

	OR $2\cos x = \dfrac{\sin 2x}{\cos 2x}$ $2\cos x.\cos 2x = \sin 2x$ $2\cos x.\cos 2x - 2\sin x.\cos x = 0$ $2\cos x(\cos 2x - \sin x) = 0$ $\cos 2x = \sin x$ or $2\cos x = 0$ $\cos 2x = \cos(90° - x)$ $x = 90°$ $2x = \pm(90° - x) + k.360°$ $k \in Z$ $3x = 90° + k.360°$ $x = 30° + k.120°$ $x = -90°$ *or* $x = 30°$ *or* $x = -90° + k.360°$ $x = -90°$	✓ $\dfrac{\sin 2x}{\cos 2x}$ ✓ $2\sin x \cos x$ ✓ $\sin x = \cos(90° - x)$ ✓ $\cos x = 0$ ✓factors ✓ equations ✓ $\pm 90°$ ✓30^0 (8)
12.3	$0° < x < 45°$ Or $-90° < x < -45°$	✓✓critical points ✓✓notation (4)
12.4	Period $= 2(360°) = 720°$	✓✓ answer (2)
12.5	$x = -45° + 25° = -20°$ $x = 45° + 25° = 70°$ **Note:** Answer only: **full marks** **OR** $2(x - 25°) = -90°$ and $2(x - 25°) = 90°$ $2x - 50° = -90°$ and $2x - 50° = 90°$ $2x = -40°$ and $2x = 140°$ $x = -20°$ and $x = 70°$	✓ $x = -20°$ ✓ $x = 70°$ (2) **[22]**

TOTAL : 150